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Multi-time period combined gas and electricity network optimisation

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Abstract

A multi-time period combined gas and electricity network optimisation model was developed. The optimisation model takes into account the varying nature of gas flows, network support facilities such as gas storage and the power ramping characteristics of electricity generation units. The combined optimisation is performed from an economic viewpoint, minimising the costs associated with gas supplies, linepack management, gas storage operation, electricity generation and load shedding. It is demonstrated on two case studies, a simple example, and on the GB network.

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1. Introduction

1.1. Overview of the gas and electricity sectors in the UK

Over the next few decades, natural gas use is projected to be the fastest growing fossil fuel in Europe, with growth being driven mainly by demand from the electricity generation sector [1, 2]. Natural gas consumption for the electricity generation sector in Europe is expected to increase by 3.7% per year from 2002 to 2030. The share of total power generation in Europe met by natural gas is forecasted to increase from 15% in 2002 to over 35% by 2030 [2].

The UK in particular has seen a dramatic rise in the number of gas fired generators, growing from 1 in 1991 to 33 in 2003 [3]. Over 33% of electrical generation capacity is now gas turbine based, with around 80% from Combined Cycle Gas Turbines (CCGT) [4, 5]. In 2005, 39% of electricity in the UK was generated from natural gas [4, 6]. By 2010, natural gas is expected to account for 46% of fuel use in electric power plants [7].

Recently, UK domestic gas production has declined with current forecasts predicting that the UK will be reliant on imports for 53% of its gas supplies by the end of the decade, rising to around 77% by 2015 [6]. This change in the source of gas supplies has led to a number of gas importation projects such as the Langeled and Balgzand-Bacton interconnectors and the Liquefied Natural Gas (LNG) importation terminals at Milford Haven and the Isle of Grain [6]. The increasing import of gas raises a number of issues, such as source dependence and the reliability of supplies from geographically remote producers, given that 40% of the world’s gas resources are located in the Caspian and Persian Gulf states [8]. Imports will result in an increase in gas transport distances and given that the gas supply infrastructure in Europe has limited alternative transport options [9], incidents such as the breakdown of gas facilities (storage, compressors, and LNG terminals) and pipelines would have a severe effect on the gas transport system.

To mitigate these low probability but high impact events the UK has started on several gas storage developments, and if all projects now planned are successfully completed the storage capacity of the UK will increase from 4.3 bcm in 2005 to over 9 bcm by 2010 [10].

1.2. Potential uses of the combined network model

In the UK, deregulation of the energy sector resulted in the separation of the vertically integrated utility and ushered in a decentralised approach to energy related decision-making. In both gas and electricity sectors, there is a separation in ownership of bulk energy transmission from other parts of the energy supply chain such as gas supply and power generation. As the UK moves into an era of increased interdependence between the gas and electricity sectors [1, 2, 11], the decentralised approach does not address the interactions between these sectors with regards to operational, security and environmental issues. It is thus useful from a strategic point of view (for asset owners and policy makers) to undertake operational and planning analysis in an integrated manner. Using supply and demand scenarios, the combined gas and electricity network optimisation model allows analysis of the following questions (Figure 1):

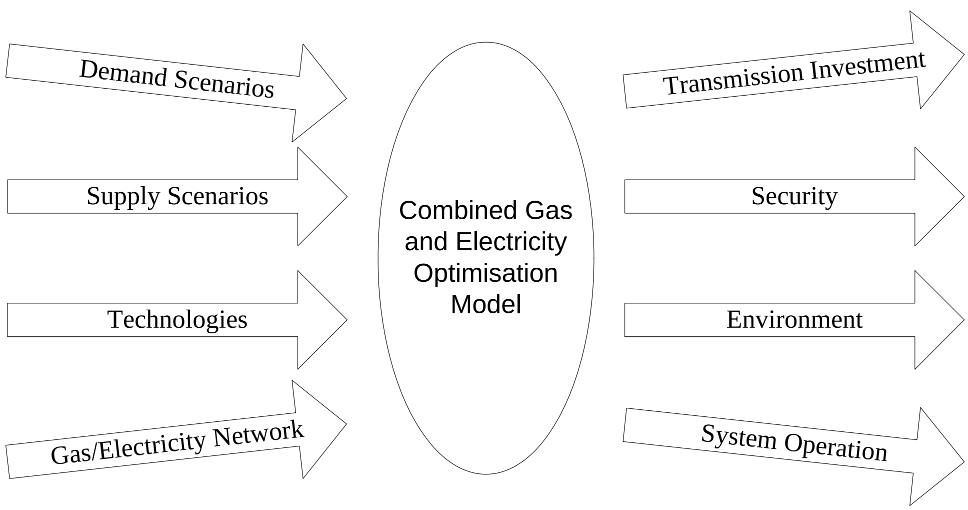

Figure 1. Potential uses of the multi-time period combined gas and electricity optimisation model

- **Transmission investment:** Within the electricity sector, network planning is closely related to generation planning. Planning in the gas sector is made on the basis of reinforcement to meet projected increases in demand and supply at existing and new locations. Combined network investment planning will allow the optimal investment plan to be determined for various supply and demand scenarios.
- **Security:** As gas is the fuel that commands the largest share of electricity generation in the UK [4,6], a combined security analysis would allow investigation into the affects of the loss of various assets or fuel source e.g. loss of a major gas terminal and the effect on the gas and electricity sectors.
- **Environment:** In the UK, the government has set an ambitious target to cut CO_2 emissions by 60% by 2050 [3]. For electricity generation the gas combustion process produces lower CO_2 emissions than coal, therefore through environmental constraints combined gas and electricity analysis will allow benefits to the environment to be calculated.
- **System operation:** Both gas and electricity networks are required to maintain a balance between supply and demand. Matching supply with demand whilst minimising costs is an operational objective for transmission system and electricity generation owners. Combined

analysis will allow the performance of the gas and electricity infrastructure to be evaluated over an operational period e.g. optimal economic loading of the gas and electricity network to meet physical constraints such as pressure and transmission limits whilst meeting demand.

Studies on the combination of the two networks have been reported in the literature. The technical and economic affects of gas and electricity networks were discussed in [12, 13]. A single time period combined natural gas and electricity optimisation model was discussed in [14, 15]. The physical characteristics of the gas and electricity network were relaxed in the multi-time period network flow model introduced in [16, 17]. This paper builds on the multi-time period concept by modelling the physical network constraints associated with gas and electricity networks. A DC power flow model represents the electricity network and in the gas network, successive time periods are linked by taking into account the volume of gas in pipes (Linepack) and important facilities such as compressors and gas storage are modelled.

2. Gas networks

The main components of a gas network are: pipelines, regulators, valves and compressors [18]. Natural gas enters the pipeline from supply sources, and is then transported to delivery points (customers) through the pipeline network. The gas flow rate in a pipe is determined by the pressure difference between upstream and downstream nodes.

2.1. Gas flow equation along a pipe

In the model, the gas flow is assumed to be one-dimensional because the change of gas properties along the radius of a pipe is negligible compared with the change along the streamline direction. The assumptions for this one-dimensional flow are [18, 19]:

- The cross-sectional area changes slowly along the path of the stream of gas;
- The radius of curvature of the pipe is large compared with its diameter;
- Temperature and velocity profiles are approximately constant along the pipe;
- The pipe is horizontal.

The gas flow along a pipe is subject to the Law of conservation of mass and Newton's second law of motion. This is described by the continuity equation (1) and the momentum equation (2) [18]:

$$\frac{\partial Q}{\partial x} = -\frac{A}{\rho ZRT}\frac{\partial p}{\partial t} \tag{1}$$

$$\frac{\partial p}{\partial x} = -\frac{\partial(\rho\omega)}{\partial t} - \frac{\partial(\rho\omega^2)}{\partial x} - \frac{2f\rho\omega^2}{D} \tag{2}$$

Where,

p : Pressure (Pascal, *Pa*)

A: Cross-sectional area of the pipe (m^2)

Q: Volumetric flow rate (m^3/s)

D: Diameter of the pipe (m)

f : Friction factor (dimensionless)

ρ : Density of gas (kg/m^3)

ω : Velocity of gas along the pipe (m^3/s)

L : Length of the pipe (m)

Z : Compressibility factor (dimensionless)

R : Gas constant ($J/Kg\,K$)

T : Temperature of gas (K)

x: Distance (m), shown in Figure 2

V : Volume of gas (m^3)

M : Mass flow rate (Kg/s)

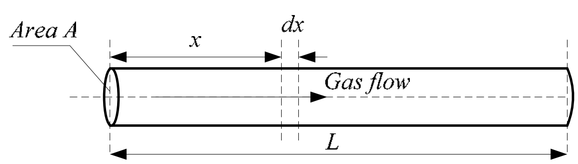

Figure 2. Gas flow along a pipe

The change in kinetic energy along a pipe due to changes in density and velocity is negligible, therefore the term $\frac{\partial(\rho\omega^2)}{\partial x}$ has little effect on the pressure drop and can be neglected [18].

Equation (2) is rewritten as:

$$\frac{\partial p}{\partial x} = -\frac{\partial(\rho\omega)}{\partial t} - \frac{2f\rho\omega^2}{D} \qquad (3)$$

The volumetric flow rate is:

$$Q = \omega A \qquad (4)$$

Substituting equation (4) into equation (3), the momentum equation becomes:

$$\frac{\partial p}{\partial x} = -\frac{\rho}{A}\frac{\partial Q}{\partial t} - \frac{2f\rho Q|Q|}{A^2 D} \qquad (5)$$

The mass flow rate is:

$$\mathrm{M} = \rho Q = \rho_n Q_n \qquad (6)$$

Where, the subscript *n* refers to the quantities at standard conditions of temperature $T_n = 288K$ and pressure $P_n = 0.1MPa$.

When considering large time steps and load conditions that do not change rapidly, the $-\frac{\rho}{A}\frac{\partial Q}{\partial t}$ term in equation (5) can be neglected [18], and therefore the continuity and momentum equations describing the transient flow of gas through a horizontal pipe are described as follows:

$$\frac{\partial Q_n}{\partial x} = -\frac{A}{\rho_n ZRT}\frac{\partial p}{\partial t} \qquad (7)$$

$$\frac{\partial p}{\partial x} = -\frac{2f\rho_n Q_n|Q|}{A^2 D} \qquad (8)$$

There are numerous equations describing the friction factor (f) relationship between pressure drop and flow rate [18]. In this paper, the Panhandle 'A' equation for high pressure networks is used (for the pressure range above 7 x 10^5 *Pa*):

$$\sqrt{\frac{1}{f}} = 6.872(\mathrm{Re})^{0.073}E \qquad (9)$$

Where Re is the Reynolds number (dimensionless), which is used to characterise gas flow conditions (for fully turbulent flow Re >> 4000, see appendix I) and *E* is the efficiency factor that accounts for extra friction and drag losses other than losses due to viscous forces.

A finite difference approximation scheme (Figure 3) is used to represent the *x* (distance) and *t* (time) derivatives for equations (7) and (8) [20].

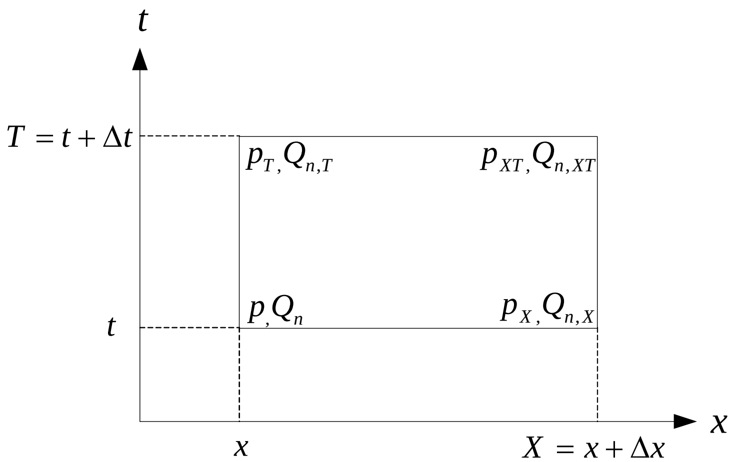

Figure 3. Finite difference cell

Where, p, Q_n, p_X and $Q_{n,X}$ are the pressure and gas flow variables at time *t*, while $p_T, Q_{n,T}, p_{XT}$ and $Q_{n,XT}$ are the variables at time $t+\Delta t$.

The steady state average pressure of a pipe is calculated as follows [18]:

At time t :

$$p_{AV} = \sqrt{\frac{1}{2}\left(p^2 + p_X^2\right)}$$

At time $t+\Delta t$:

$$p_{TAV} = \sqrt{\frac{1}{2}\left(p_T^2 + p_{XT}^2\right)}$$

The average gas flow in a pipe at time $t+\Delta t$ is:

$$Q_{n,TAV} = \frac{1}{2}\left(Q_{n,T} + Q_{n,XT}\right)$$

The gas equation of state is [18]:

$$ZR = \frac{p_n}{\rho_n T_n} = \frac{p}{\rho T} \qquad (10)$$

The finite element scheme allows the original partial differential equations (7, 8) to be transformed into ordinary differential equations [20]:

$$\frac{Q_{n,XT} - Q_{n,T}}{\Delta x} = -\frac{A}{\rho_n ZRT}\frac{\left(p_{TAV} - p_{AV}\right)}{\Delta t} \qquad (11)$$

$$\frac{p_{XT} - p_T}{\Delta x} = -\frac{2ZRTf\rho_n^2\left(Q_{n,TAV}\right)\left|Q_{n,TAV}\right|}{A^2 D p_{TAV}} \qquad (12)$$

2.2. Linepack modelling

The Linepack of a pipe is the volume of gas stored in the pipe and is a key factor that affects the ability to supply gas to demand nodes i.e. a highly packed pipe allows fluctuations in demand to met locally as gas supply from a distant source will take time (typically hours) to reach its intended destination.

Using Boyle's law:

$$p_{\mathrm{AV}}\mathrm{V} = p_{\mathrm{n}}V_n \qquad (13)$$

Therefore, the Linepack (LP) of a gas pipe is calculated by combining equations (10) and (13):

$$LP = V_n = \frac{p_{AV}V}{\rho_n ZRT_n} \qquad (14)$$

This equation is suitable for calculating the volume of gas in a pipe when the gas flow is in steady state.

Equation (14) illustrates that the pipe linepack is proportional to the average pressure in the pipe section, therefore, increasing the average pressure of the pipe will increase the linepack and vice versa.

Under dynamic situations, the gas flow into and out of a pipe fluctuates with changing supply and demand. According to the Law of conservation of mass, the change of total gas volume is equal to the difference between the flow into and out of the pipe. Thus, we have:

$$LP(t) = LP_0 + \int_0^t (Q_n - Q_{n,X})dt \qquad (15)$$

Where, LP_0 is the initial gas stored in the pipe and is calculated by equation (14) in the steady state condition.

2.3. Non-pipe elements

For gas networks, the most important non-pipe elements are compressors and storage facilities.

2.3.1. Compressors

The power required from the compressor prime-mover is calculated by equation (16) [18]:

$$P^C = \frac{Q_n^c \alpha}{\eta(\alpha - 1)}\left[\left(\frac{p^o}{p^{in}}\right)^{(\alpha-1)/\alpha} - 1\right] \qquad (16)$$

Where,

P^C : Compressor power ($10^5 W$)
p^o, p^{in}: Outlet and inlet pressures (Pa)
Q_n^c : Flow rate through the compressor at standard conditions (m^3/s)
η : Overall compressor efficiency
α : Polytropic exponent [18]

In practice, the restrictions on a compressor are reduced to the following [18]:

$$1 < \frac{p^o}{p^{in}} < S_{\max} \quad \left(\frac{p^o}{p^{in}} = S \Rightarrow \text{Compressor pressure ratio} \right) \qquad (17)$$

$$Q_n^c \leq Q_{n,Max}^c \qquad (18)$$

$$p^o \leq p^{CompMax} \qquad (19)$$

$$p^{in} \geq p^{CompMin} \qquad (20)$$

$$P^C \leq P^{C,Max} \qquad (21)$$

Where, $p^{CompMax}$ and $p^{CompMin}$ are the maximum and minimum compressor pressures. $Q_{n,Max}^c$ is the maximum compressor flow rate and $P^{C,Max}$ is the maximum compressor power.

The amount of gas tapped by the compressor is approximated by [14]:

$$\tau^c = \beta \, P^C \qquad (22)$$

Where,

τ^c : Amount of gas tapped by a compressor (m^3/s)

β : Gas turbine fuel rate coefficient of a compressor

2.3.2. Gas storage modelling

Gas storage provides a potential substitute gas source if supply from a terminal or elsewhere is disrupted. Storage facilities are represented by the following characteristics [21]:

***Working gas*:** This is the useful volume of gas that can be withdrawn from storage. The actual total volume of gas in storage is a summation of the working gas and cushion gas. Cushion gas is the volume of gas required in storage to maintain an adequate pressure, and is not normally used.

$$S_s^{Cush} \leq S_{s,t}^{Work} \leq S_s^{Max} \qquad (23)$$

Where,

$S_{s,t}^{Work}$: (Working) Storage volume of facility *s* at time *t* (m^3)

S_s^{Cush} : Cushion gas capacity of storage facility *s* (m^3)

S_s^{Max} : Gas storage capacity of storage facility *s* (m^3)

$Q_{s,t}^{MaxWithdrawal}$: Maximum gas withdrawal rate of storage facility *s* at time *t* (m^3/s)

$Q_{s,t}^{Max\,Injection}$: Maximum gas injection rate of storage facility *s* at time *t* (m^3/s)

$Q_{s,t}$: Gas storage flow rate of storage facility *s* at time *t* (m^3/s)

K_s, K_s^1, K_s^2 : Gas storage coefficients for storage facility *s*. Coefficient calculation is dependant on the surface area of storage opening, base storage capacity, and proportionality constants [21].

Therefore, for each time step, storage capacity is calculated as follows:

$$S_{s,t+1}^{Work} = S_{s,t}^{Work} + Q_{s,t} \qquad (24)$$

Where, $Q_{s,t}$ determines the direction of gas storage flow:

$Q_{s,t}^{MaxWithdrawal} \geq Q_{s,t} > 0$ Withdrawing gas from storage facility *s* at time *t* (25)

$-Q_{s,t}^{Max\,Injection} \leq Q_{s,t} < 0$ Injecting gas into storage facility *s* at time *t* (26)

***Gas withdrawal*: ** This is at its highest when a storage facility is close to its maximum capacity and lowest when nearly empty [21]:

$$Q_{s,t}^{Max\,Withdrawal} = K_s \sqrt{S_{s,t}^{Work}} \qquad (27)$$

Maximum withdrawal occurs at maximum gas capacity S_s^{Max} therefore K_s from equation (27) can be calculated.

***Gas injection*: ** The injection rate is at its lowest when a storage facility is at maximum capacity and at its highest when storage is empty [21].

$$Q_{s,t}^{Max\,Injection} = -K_s^1 \sqrt{\frac{1}{S_{s,t}^{Work} + S_s^{Cush}} + K_s^2} \qquad (28)$$

When a storage facility is at maximum capacity, no more gas injection can take place. Gas injection is therefore zero, $S_{s,t}^{Work}$ and S_s^{Cush} are known, hence K_s^2 from equation (28) can be calculated.

The maximum gas injection rate occurs when $S_{s,t}^{Work}$ equals zero, therefore K_s^1 from equation (28) can be calculated.

2.4. Gas network constraints

At each node in the gas network, gas flow balance and pressure constraints were imposed.

For each time step, gas inflows at each node (gas supply, gas storage withdrawal) are balanced with gas outflows (gas demand, compressor fuel usage, gas storage injection):

$$M_u Q^{supp} + M_p Q_n + M_c Q_n^c - M_t \tau^c + M_s Q_s = M_d \left(Q^{Gasdem} - Q^{GasShed}\right) \qquad (29)$$

$$p^{\min} \leq p \leq p^{\max} \qquad (30)$$

Where,

$p^{\min}, p^{\max}$: The lower and upper pressure bounds

M_u : Node-supply incidence matrix

M_p : Node-pipe flow incidence matrix

M_c : Node-compressor incidence matrix

M_t : Node-compressor fuel tap incidence matrix

M_s : Node-storage incidence matrix

M_d : Node-load incidence matrix

$Q^{GasShed}$: Gas load shedding (m^3/s)

Q^{GasDem} : Gas demand (m^3/s)

3. Electricity networks

A DC power flow model [22, 23] was used to represent the electricity network. The DC power flow formulation enables the calculation of MW power flows in each individual transmission line. The DC power flow model is a simplification of an AC power flow and is based on the following assumptions:

- The line resistance in a high voltage transmission system is very much smaller when compared to line reactance, such that resistance and system losses can be neglected.
- The phase voltage angle difference of a high voltage line is very small.
- The bus voltage per unit is close to nominal value (~1.0).

3.1. Power balance constraints

The power balance equations were satisfied such that total generation is equal to total demand minus load shedding at each time step (equation 31):

$$\sum_{i}^{G} P_{i,t}^{Gen} = \sum_{j}^{Dd} P_{j,t}^{Demand} - \sum_{j}^{Dd} P_{j,t}^{ElecShed} \quad \begin{array}{l} \text{for} \quad i = 1,2,...,G \ \text{(number of generators)} \\ \text{for} \quad j = 1,2,...,Dd \ \text{(number of load buses)} \end{array} \qquad (31)$$

Where, $P_{i,t}^{Gen}$ is the power output of generation unit *i* at time *t*, $P_{j,t}^{Demand}$ and $P_{j,t}^{ElecShed}$ are the demand and load shedding at bus *j* at time *t*.

3.2. Generation constraints

The generation schedule produced was kept within the physical limitations of the generating units (equation 32):

$$P_i^{Gen(\min)} \le P_{i,t}^{Gen} \le P_i^{Gen(\max)} \qquad \text{for} \quad i = 1,2,...,G \qquad (32)$$

3.3. Power flow sensitivity constraints

To maintain system security the power flowing in each transmission line was maintained within maximum power flow limits (equation 33):

$$-F_l^{\max} \le F_{l,t}^0 + \sum_{j=1}^{Dd} h_{lj}\left(P_{j,t}^{Gen,A} - P_{j,t}^{Gen,0}\right) \le F_l^{\max} \qquad \text{for each transmission line } l \qquad (33)$$

Where,

$P_{j,t}^{Gen,0}$: Initial power generation at bus *j* at time *t*

$P_{j,t}^{Gen,A}$: Power generation at bus *j* needed to avoid violation on line l at time *t*

h_{lj} : Linear sensitivity factors for line l corresponding to generation at bus j (Sensitivity factors are used to calculate overloads in transmission lines. These factors show the approximate change in the power flow for line l to a change in power generation at bus j [22, 23])

$F_{l,t}^0$: Initial power flow that violates the limit for line l at time *t*

$F_l^{\max}$: Maximum permitted power flow limit for line l

3.4. Generator ramp up/down modelling

Since power plants cannot ramp up or ramp down instantaneously, the following constraints were imposed:

Ramping up:

$$P_{i,t}^{Gen} - P_{i,t-1}^{Gen} \leq RU_i \qquad (34)$$

Ramping down:

$$P_{i,t}^{Gen} - P_{i,t-1}^{Gen} \geq RD_i \qquad (35)$$

Where, RU_i and RD_i represents the maximum ramp up and down power output for generation unit *i*.

4. Combined gas and electricity modelling

Gas turbine generators provide the linkage between gas and electricity networks. They are considered as energy converters between these two networks. For the gas network, the gas turbine was looked upon as a gas load. Its value depends on the power flow in the electricity network. In the electricity network, the gas turbine generator is a source.

The relationship between the gas fuel flow and the real electrical power generated is expressed as:

$$P^{Gen} = \varphi Q_n H_g \qquad (36)$$

Where,
φ : Thermal efficiency of the gas turbine
H_g : Gas heating value (~39 MJ/m^3)

5. Multi-time period optimisation of combined networks

For the combined gas and electricity network, the objective is to minimise the combined operational costs whilst meeting demand requirements over the entire time horizon. The objective function is useful for system operators and market participants wanting to drive down costs associated with gas supplies, linepack management, gas storage operation, power generation, and load shedding.

5.1. Objective function

$$Min\ \ Total(£)_{gas+elec} = \sum_{t=1}^{Horizon}\left(\begin{array}{l}\sum_{a=1}^{NQ} SP_t\, Q_{a,t}^{Supp} + \sum_{k=1}^{NL} SP_t\, dLP_{k,t} + \sum_{s=1}^{NS}\left(-Q_{s,t} IC_{s,t} + Q_{s,t} WC_{s,t}\right) + \\ \sum_{b=1}^{NDG} C_b Q_{b,t}^{GasShed} + \sum_{i=1}^{NG} C_{i,t} P_{i,t}^{Gen} + \sum_{j=1}^{NDE} C_j P_{j,t}^{ElecShed}\end{array}\right) \quad (37)$$

All units are converted to their hourly equivalents.

Where,

NQ : Number of gas supply nodes

NL : Number of gas pipes

NS : Number of storage facilities

NG : Number of power generation plants

NGE : Number of electricity load buses

NGG : Number of gas load nodes

SP_t : Gas price at time *t* ($£/m^3$)

$Q_{a,t}^{Supp}$: Gas flow from source node *a* at time *t* (m^3/h)

$dLP_{k,t}$: Rate of change of gas linepack of pipe *k* at time *t* (m^3/h)

$Q_{s,t}$: Gas storage flow rate of storage facility *s* at time *t* (m^3/h)

Gas injection → *–ve*

Gas withdrawal → *+ve*

$P_{j,t}^{ElecShed}$: Electrical load shedding at bus *j* at time *t* (*MW*)

$Q_{b,t}^{GasShed}$: Gas load shedding at node *b* at time *t* (m^3/h)

$IC_{s,t}$: Storage injection cost for facility *s* at time *t* ($£/m^3$)

$WC_{s,t}$: Storage withdrawal cost for facility *s* at time *t* ($£/m^3$)

$C_{i,t}$: Cost of power generator *i* at time *t* ($£/MWh$)

C_j : Cost of electricity load shedding at bus *j* ($£/MWh$)

C_b : Cost of gas load shedding at gas node *b* ($£/m^3$)

The objective function (equation 37) is subject to gas and electricity network constraints.

6. Case studies

The multi-time period combined gas and electricity model was formulated using the Dash Xpress optimisation suite. The Xpress-SLP (Sequential Linear Programming) solver for non-linear programming was used to minimise the objective function over the entire time horizon.

To aid convergence and feasibility the following procedures were adopted for the optimisation process:

Feasible starting point: all decision variables were given initial values corresponding to a single time period steady state optimisation run of the model.

Scaling: decision variables where scaled so that numerical values do not vary over many orders of magnitude

The optimisation was performed on two case studies: the simple example network (Figure 4) and the GB gas and electricity network [5, 6] (Figures 12 and 13 in appendix IV).

6.1. Optimisation of simple network

The simple example consisted of four pipes, one compressor, and a storage facility (no gas in storage at the start of the optimisation). The electricity network has one transmission line connected to two busbars. Three power plants are connected to this network, a CCGT (G) and coal plant (C1) at busbar A and a further coal plant (C2) at busbar B (see appendix II for parameter values).

For all results, pressure is represented in bars (1×10^5 *Pascals* = 1 *bar*) and gas flow rates are calculated on an hourly basis ($m^3 / h = cm / h$). The optimisation of the simple network has an hourly time-step.

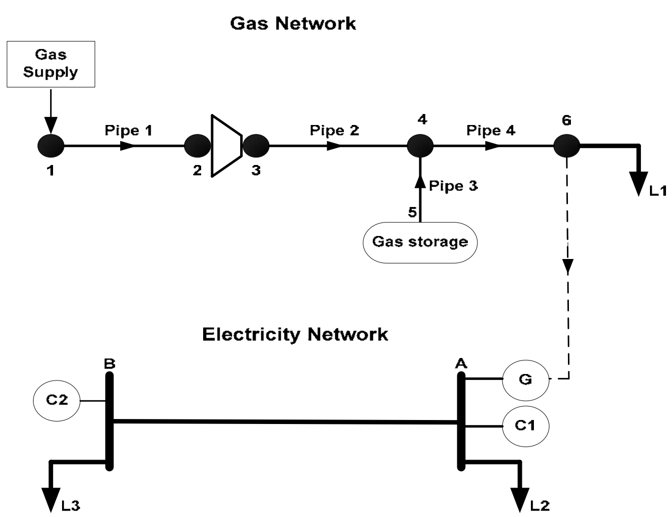

Figure 4. Simple combined gas and electricity network

Figure 5 shows the gas/electrical demand and gas prices over a 24-hour period for the simple example network.

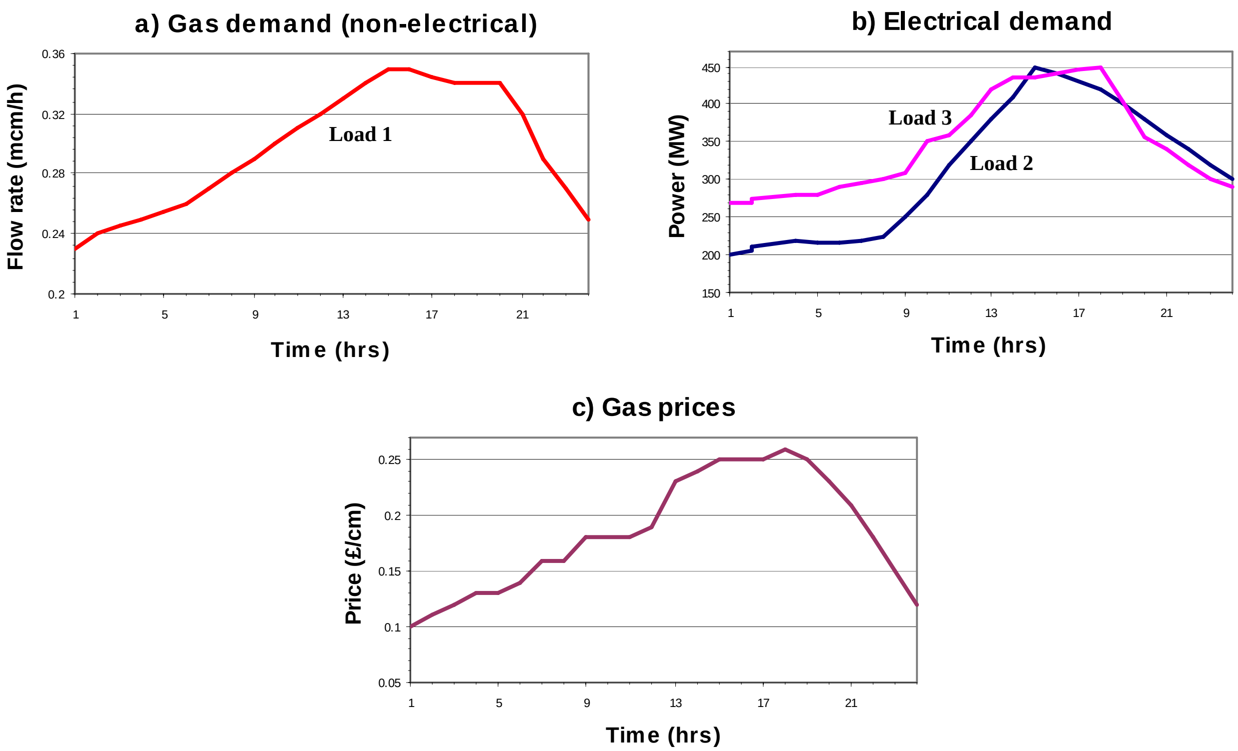

Figure 5. Gas/electrical demand and gas prices

Case A: reference case-

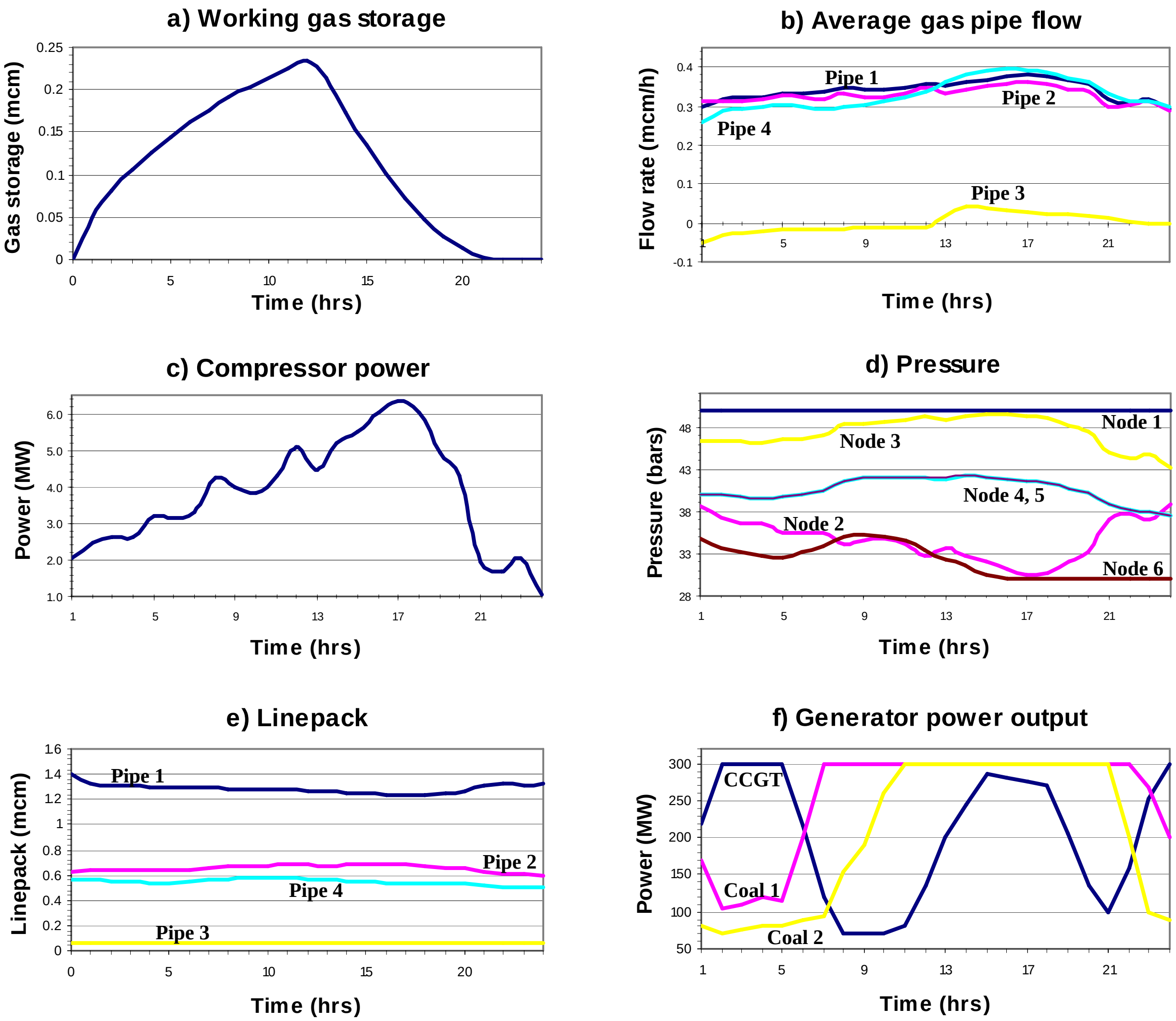

Figure 6. Reference case optimisation results

The optimisation for this case illustrates that the gas storage facility stores gas during low price periods (Figures 5.c and 6.a) and utilizes the stored gas when demand (Figures 5.a and 5.b) and gas prices increase. The gas storage facility shows its ability to provide network support, in this case, gas is withdrawn from storage at hour 13 into pipe 3 (Figure 6.b, positive gas flow for pipe 3 indicates gas withdrawal from storage) to satisfy increased electrical and non-electrical gas demand. During hours 10-16, gas demand increases, Figure 6.c shows compressor usage increasing and thus providing pressure support at node 3 (Figure 6.d) and helping downstream pressure nodes to stay within operating limits. The Linepack of the gas network is shown in Figure 6.e, and is used to alleviate supply and demand fluctuations in the network. For instance, during hours 18 to 21 compressor usage decreases, resulting in a drop in average pressure for all downstream pipes (Figure 6.d, average pressure between nodes 3-4 and 4-6). The drop in average pressure leads to a reduction in the linepack of pipes 2 and 4 shown in Figure 6.e in order to satisfy gas demand.

The gas-fired power plant (CCGT) will only operate when conditions are favourable such as low gas prices (the total price of gas includes not only the gas price at the terminal but also the use of gas facilities such as storage and compressors) and during periods when there is a lack generation capacity to satisfy demand. Figure 6.f shows the CCGT unit operating at maximum capacity during hours 2 – 5, this coincides with low gas prices. As gas prices increase both coal units eventually become cheaper for generating electricity and the CCGT unit is ramped down during hours 5 – 8. During hours 10 – 16 the CCGT unit, despite being expensive, is required to increase generation to help satisfy increasing electrical demand as both coal units are operating at maximum capacity. The CCGT unit is ramped up during hours 21-24 due to gas prices falling and the coal units becoming more expensive to operate.

Case B: Gas storage outage-

In this case, the gas storage facility breaks downs at hour seven and stays offline. The optimisation approach is to assume perfect foresight therefore the optimiser will be aware of the failure of the storage facility at hour 7 at the start of the optimisation process.

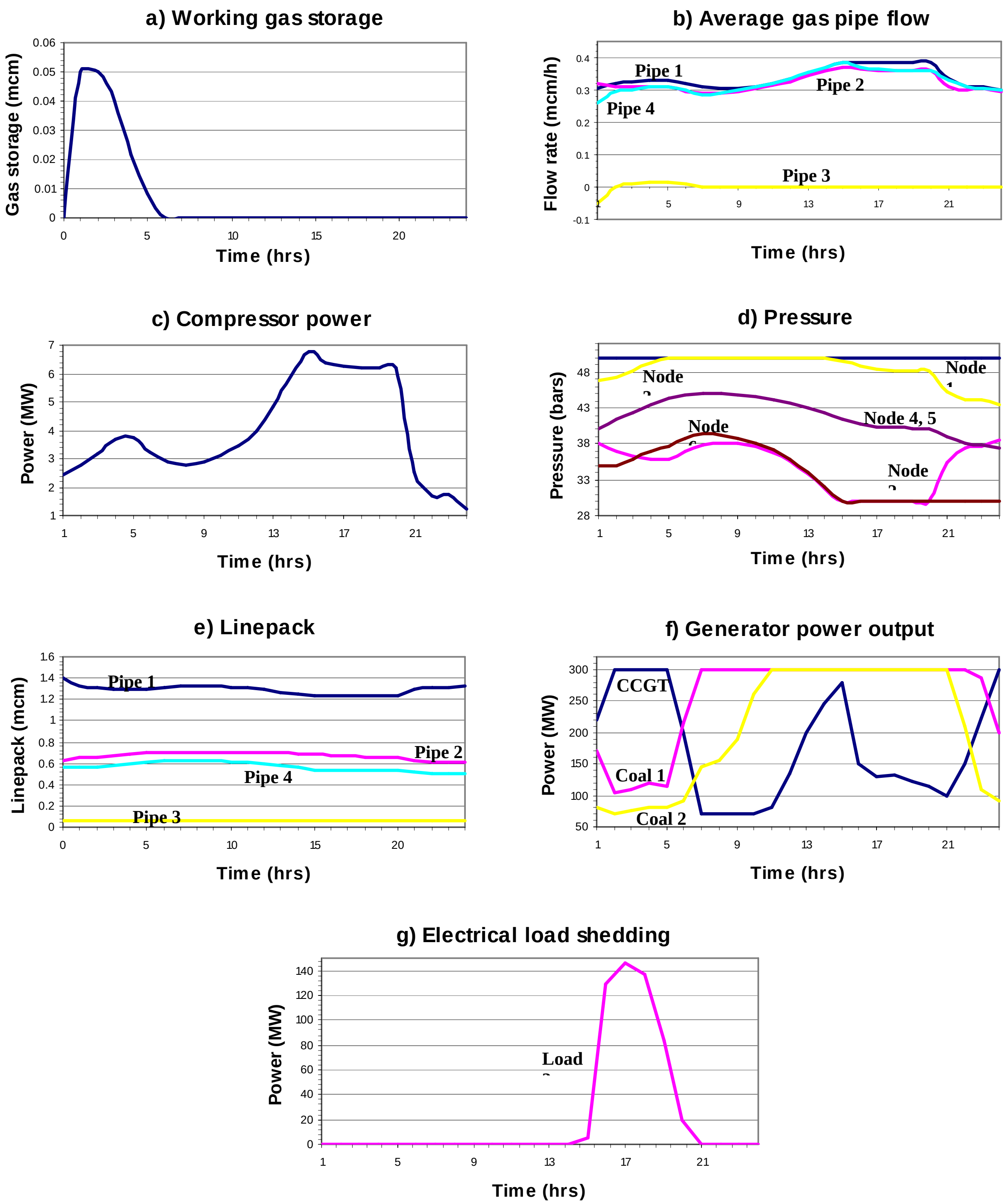

Figure 7. Gas storage outage optimisation results

This case shows the value of an integrated approach to the analysis. Figure 7.g shows that the loss of the storage facility has resulted in electrical shedding at busbar B (due to the cost of shedding being higher at busbar A). During hours 13-20, Figure 7.c shows an increase in compressor activity from the reference case in order to maintain the pressure of downstream pipes within operating limits. However, the compressor is unable to prevent electrical load shedding from occurring as Figure 7.b shows pipe 1 (supply pipe to the rest of the network) delivering gas supplies at it maximum capacity between hours 15 to 20. The linepack of the gas system shown in Figure 7.e is also unable to provide much support without breaching minimum pressure constraints. As the cost of shedding for non-electrical gas demand is very high, the generation output from the CCGT unit is driven down to levels that the gas system can support. Since there is now a lack of generation capacity to satisfy electrical demand despite coal-fired units running at maximum capacity there is no option but to commence the shedding process at busbar B.

The objective for the two cases was to minimise the total cost of operating the combined network (gas supplies, storage operation, generation costs, and load shedding) whilst satisfying demand and physical constraints in both networks. The cost of operating the combined network for cases A and B over the 24 hours are £1.92M and £2.44M respectively. There is a clear difference between the costs for cases A and B, and therefore a case could be made for further strengthening of the gas and/or electricity infrastructures such as additional storage facilities.

6.2. Optimisation of combined GB network

For the GB gas network, gas flow rates are calculated on a daily basis ($m^3/d = cm/d$). The optimisation of the GB network has a daily time-step. Figure 8 shows the GB gas/electrical demand and gas prices for the month of December 2005 [24, 25].

All gas storage facilities in the GB network are at maximum capacity at the start of the optimisation. For GB gas and electricity parameter values, see Appendix III.

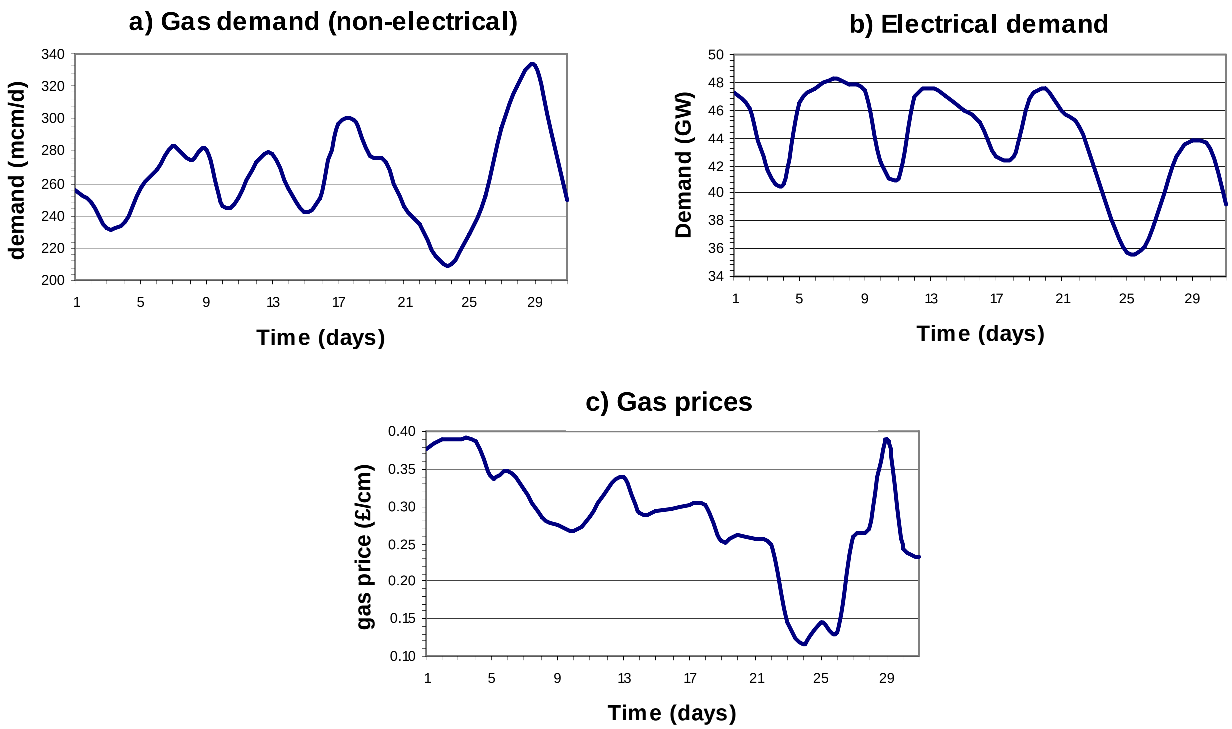

Figure 8. GB gas/electrical demand and gas prices

Case A: GB reference case-

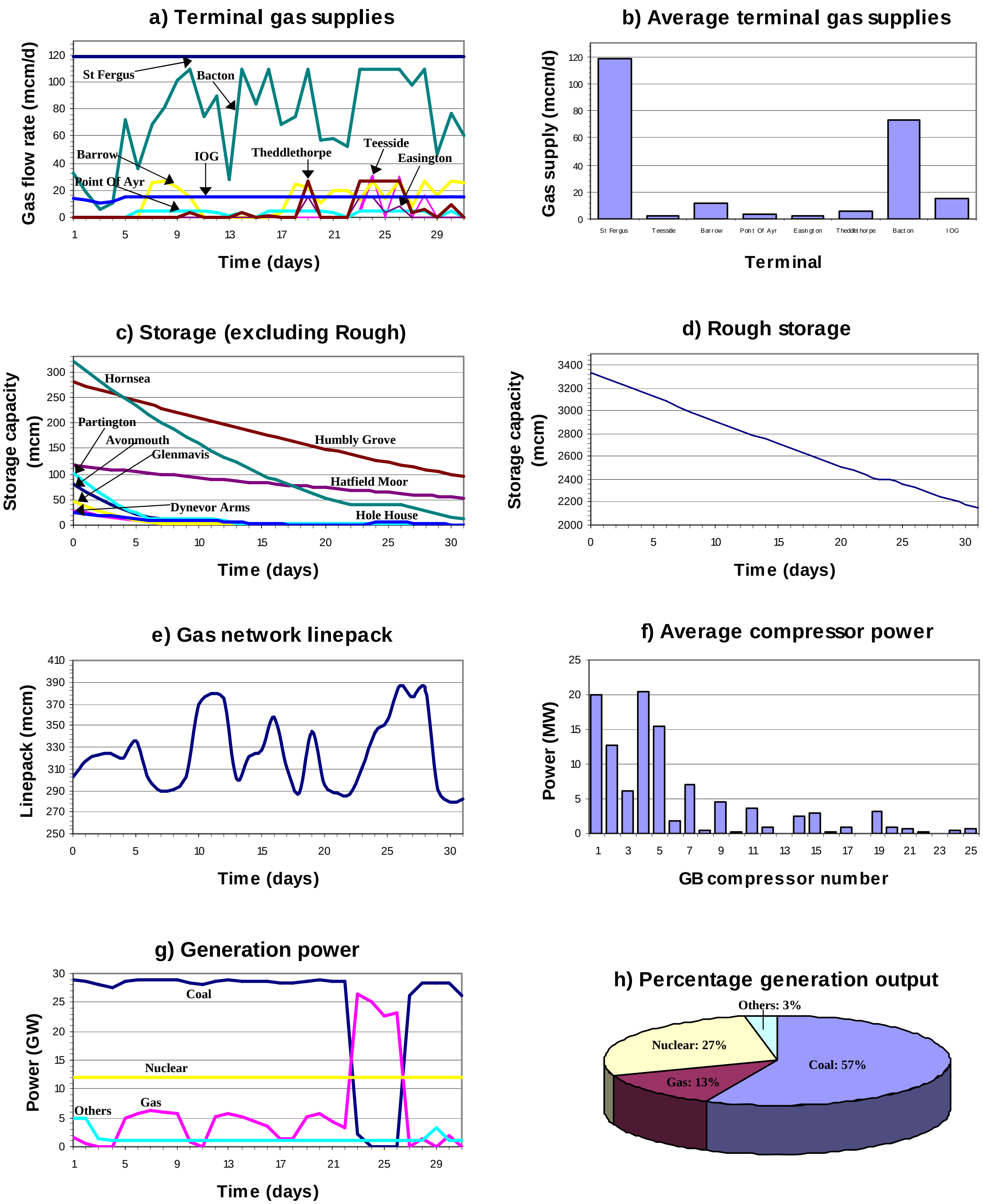

Figure 9. Reference GB case optimisation results

The St. Fergus and Bacton terminals are shown to be the largest contributors of gas supplies over the month (Figures 9.a and 9.b). The St Fergus terminal has the largest capacity for potential gas supplies followed by Bacton. The price of gas at each terminal is the same with the exception of the St. Fergus terminal where a 5% discount is applied. Gas transmission from the St. Fergus terminal to the rest of the network requires extensive compressor use in Scotland in order to maintain a north to south flow (Figure 9.f, compressors 1-5). The gas network linepack shown in Figure 9.e is proportional to the average pressure in the network (linepack in a network decreases when gas demand increases, if gas supply is held constant). During days 24-26, the network linepack is shown to increase as gas demand increases this is due to the linepack being supported by gas supplies from primarily the Bacton and Teeside terminals. The storage facilities (Figures 9.c and 9.d) are all withdrawing gas near their maximum flow rates to satisfy gas demand and support network linepack (pressure). The Rough storage facility has contributed over 1200 mcm of gas supplies over the month.

The power generation output of different technologies over the month is shown in Figure 9.g. Given that nuclear, wind and hydro are must run units and all generating at their maximum capacities, it is left to coal-fired generation as the next cheapest generation technology to provide for the bulk of electricity demand. Figure 9.h shows that gas-fired generation contributed 13% of total generation output over the month. During high gas price days, gas-fired generation is only used when coal-fired generation assets are fully utilized. Gas-fired generation is at is it highest during days 23-26, this is when gas prices are low and thus gas displaces coal as the dominant generation fuel.

Case B: Loss of Bacton terminal-

In this case, the optimisation is run without the Bacton terminal.

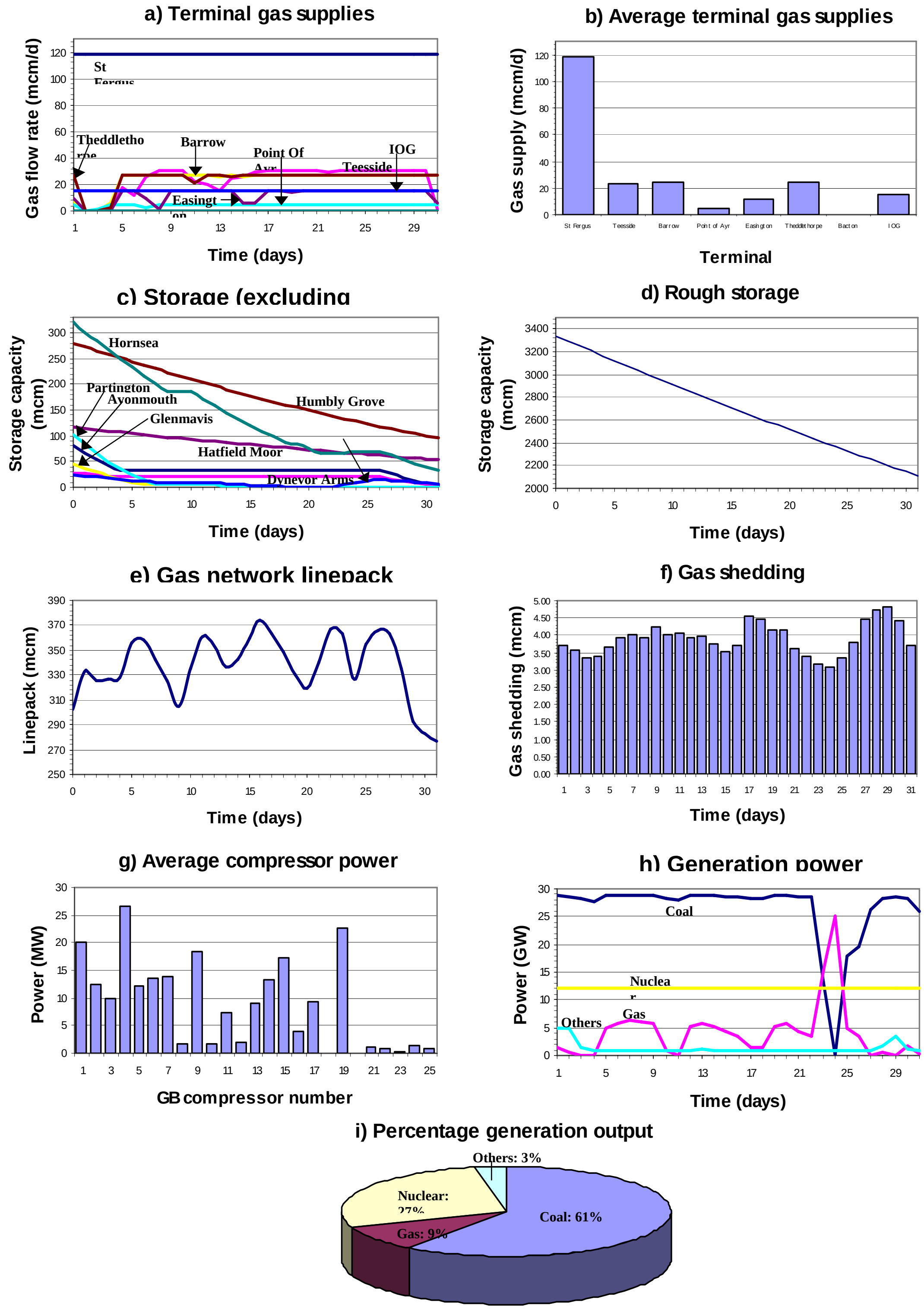

Figure 10. Loss of Bacton terminal optimisation results

The loss of Bacton has resulted in the other terminals (Teesside, Barrow, Point Of Ayr, Theddlethorpe and Easington) to supply gas at near their maximum capacities during most of the month (Figures 10.a and 10.b). Compressor use (Figure 10.g) is continuing to provide pressure support within the vicinity of the St Fergus terminal and there is an increase in compressor usage located close to other terminals. Figure 10.f shows that gas shedding (non-electrical) has taken place throughout the month. Gas shedding is mainly attributable to the interruption of gas supplies to load nodes reliant on the Bacton terminal and in order to keep the network linepack (pressure) within acceptable levels.

The power generation output shown in Figure 10.h is relatively unchanged from the reference case, apart from days 23-27 where due to pressure constraints in the gas network, gas delivery to gas-fired generation units is constrained this is despite low gas prices and relatively low gas demand during this period. Consequently, gas use for electricity generation over the month is down from 13% in the reference case to 9% (Figure 10.i).

Case C: Loss of Bacton and Rough storage facility-

In this case, the optimisation is run without Bacton terminal and Rough storage facility.

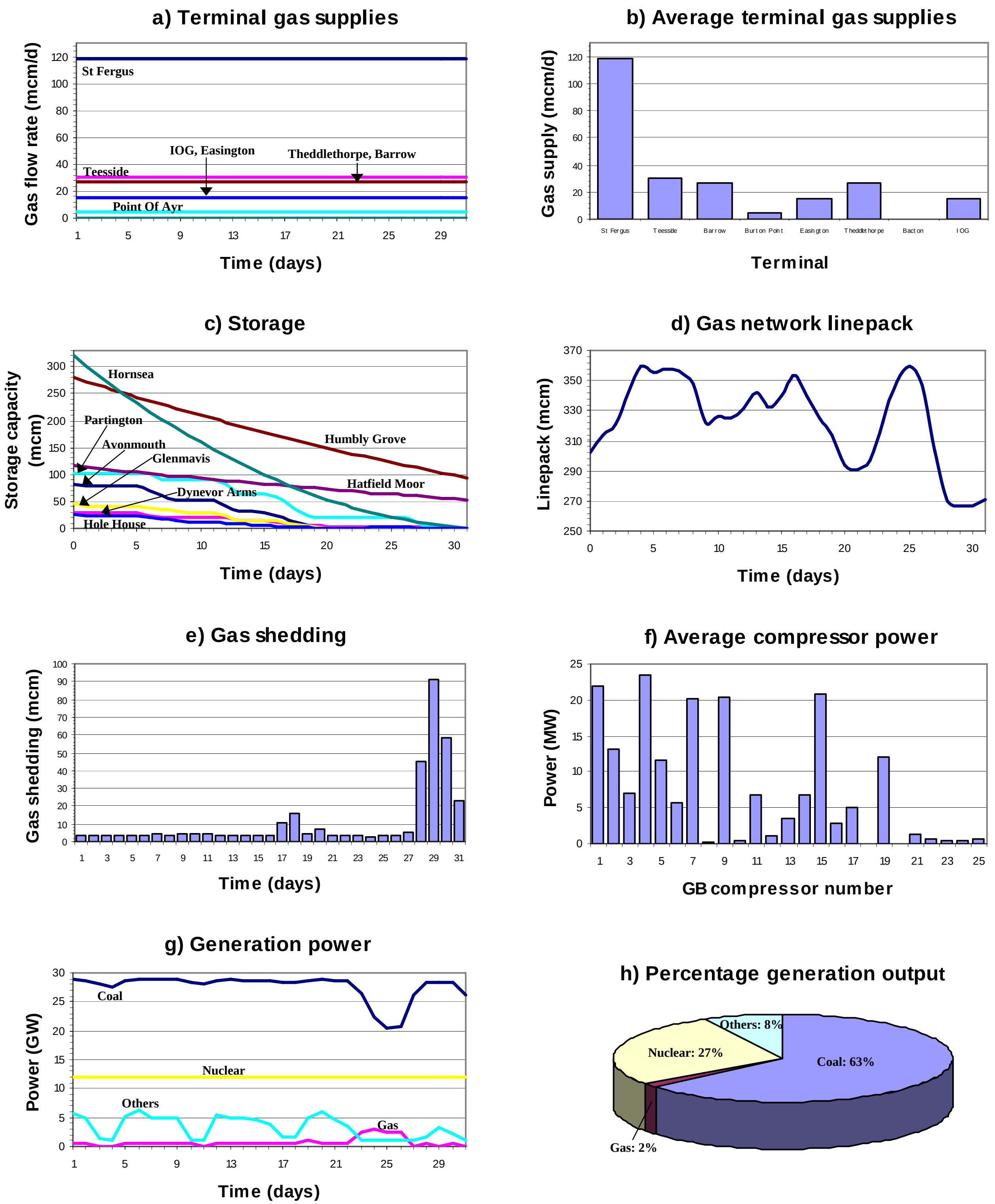

Figure 11. Loss of Bacton terminal and Rough storage facility optimisation results

All gas terminals in this case are supplying gas at their maximum capacities (Figures 11.a. and 11b). Figure 11.c shows the storage facilities (excluding Rough) are delivering gas in a similar fashion to the reference case. The loss of the Rough storage facility has exacerbated the supply situation, therefore to support network linepack (Figure 11.d), substantial non-electrical gas shedding (Figure 11.e) was necessary during high gas demand periods (days 28-30, Figure 8.a).

Generation power output in Figure 11.g shows that gas-fired generation units are operating at the margins due to pressure constraints in the gas network. Coal is the dominant fuel for electricity generation with other fuels such as oil used when extra generation capacity is required despite being more expensive than gas-fired generation. Figure 11.h shows gas use for electricity generation over the month has decreased to 2%.

The decreased use of gas-fired generation and substantial non-electrical gas shedding has underlined the importance of large-scale gas storage facilities in providing backup gas supplies to both networks.

The cost of operating the combined network over the month for cases A, B, and C are £2.7, £ 4.2, and £6.7 billion respectively. There is a large difference between the costs for these cases and is mainly attributable to the very high cost of shedding. The high costs imposed on shedding in the gas network is there to discourage the shedding of non-electrical gas demand and reflects the cost and time taken to disconnect and reconnect loads to the network.

7. Conclusions

A multi-time period combined gas and electricity network optimisation model was described in this paper. The combined network model consists of a DC load flow model for the electricity network and a detailed model of the gas network that accounts for the varying nature of gas flows along with gas support facilities such as gas storage and compressor stations. The model links the two networks through gas turbine generators connected to both networks. The combined gas and electricity optimisation model is formulated using the Dash Xpress optimisation suite. The Xpress-SLP non-linear solver is used to minimise costs relating to gas supplies, storage operation, power generation, and load shedding over the entire time horizon. The combined optimisation model was demonstrated on two case studies, a simple example network, and on the GB network.

The simple example network has illustrated the significance of gas storage to both networks. The loss of the gas storage facility led to electrical load shedding due to gas pressure constraints restricting the utilization of gas-fired generation.

The GB network model included all the current generation units connected to the electricity network and all the major gas terminals, pipes, compressors, and storage facilities in the gas network. The combined GB network optimisation of an outage at the Bacton gas terminal illustrated the importance of having alternative gas supplies from a number of gas supply terminals for entry into the pipe network. With the loss of the Rough storage facility, gas load shedding was greatly increased and thus indicated the importance of this facility in mitigating the loss of the Bacton terminal.

Through the development of scenarios that envisage the loss of key facilities, the combined gas and electricity optimisation model has shown itself to be valuable for assessing the consequences of failure to vital facilities on the combined network.

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Appendix I: Friction factor calculation

The Reynolds number is described by equation (A1):

$$Re = \frac{4\rho_n Q_n}{\mu D \pi} \qquad \text{(A1)}$$

Where, μ is the viscosity of gas and is assumed to be constant (3.77x10^{-5} Ns/m)

Substituting equation (A1) into the friction factor equation (9) gives the following:

$$\sqrt{\frac{1}{f}} = 6.872\left(\frac{4\rho_n Q_n}{\mu D \pi}\right)^{0.073} E \qquad \text{(A2)}$$

(Efficiency factor *E* = 0.95)

The specific gravity S_g (0.589 is used) of gas is defined as a ratio of the density of gas to that of air $(\rho_{n,air} = 1.236\ kg/m^3)$ at standard conditions:

$$S_g = \frac{\rho_n}{\rho_{n,air}} \qquad \text{(A3)}$$

Substituting equation (A3) into equation (A2) gives:

$$\sqrt{\frac{1}{f}} = 14.94\left(\frac{S_g Q_n}{D}\right)^{0.073} E \qquad \text{(A4)}$$

Appendix II: Example system data

Table 1. Line data of the electricity network

Line, bus to bus	X (*Per Unit*)	Max Tx (*MW*)
1, A-B	0.2	200

Base 100MVA

Table 2. Generator data of the electricity network

Bus	Plant Type	Minimum Generation (*MW*)	Maximum Generation (*MW*)	Ramp-Up (*h*)	Ramp-down (*h*)
A	CCGT 1	70	300	2	2
A	Coal 1	70	300	3	3
B	Coal 2	70	300	3	3

Table 3. Demand data of the gas and electricity network

Time(*h*)	Load L1 (*cm/h*)	Load L2 (*MW*)	Load L3 (*MW*)
1	230000	200	270
2	240000	205	270
3	245000	210	275
4	250000	220	280
5	255000	215	280
6	260000	215	290
7	270000	220	295
8	280000	225	300
9	290000	250	310
10	300000	280	350
11	310000	320	360
12	320000	350	385
13	330000	380	420
14	340000	410	435
15	350000	450	435
16	350000	440	440
17	345000	430	445
18	340000	420	450
19	340000	400	405
20	340000	380	355
21	320000	360	340
22	290000	340	320
23	270000	320	300
24	250000	300	290

Table 4. Pipe data of the gas network

Pipe	Sending node	Receiving node	Length (m)	Diameter (mm)
1	1	2	100,000	600
2	3	4	50,000	600
3	5	4	5,000	600
4	4	6	50,000	600

For each pipe: Efficiency (E) = 0.92, Temp = 288K, Min pressure = 30 bars, Max pressure = 50 bars, Compressibility factor (Z) = 0.95

Table 5. Compressor data

Item	Lower bound	Upper bound
Compressor pressure ratio	1	4

For each compressor: Efficiency = 0.8, Polytropic exponent = 1.27, Beta = 0.003, Maximum power = 10MW

Table 6. Cost of shedding

Load	Cost
Electrical load shedding at busbar A	1050(*£/MWh*)
Electrical load shedding at busbar B	1000(*£/MWh*)
Gas load shedding	100 (*£/cm*)

Table 7. Cost of generation

Generation unit	Cost (*£/MWh*)
CCGT	Dependant on gas price
Coal 1	32
Coal 2	35

Table 8. Gas storage costs

Item	Cost (*£/cm*)
Storage withdrawal costs	0.01
Storage injection costs	0.01

Table 9. Gas storage data

Node	Storage Type	Max withdrawal (*cm/h*)	Max injection (*cm/h*)	Working capacity (*cm*)	Cushion capacity (*cm*)
5	Salt cavern	50000	50000	300000	40000

Table 10. Gas price data

Time(h)	**Gas price (S1) (*£/cm*)**
1	0.1
2	0.11
3	0.12
4	0.13
5	0.13
6	0.14
7	0.16
8	0.16
9	0.18
10	0.18
11	0.18
12	0.19
13	0.23
14	0.24
15	0.25
16	0.25
17	0.25
18	0.26
19	0.25
20	0.23
21	0.21
22	0.18
23	0.15
24	0.12

Appendix III: GB system data

GB gas and electricity demand data available online [24].

Table 11. Cost of power generation and installed capacity

Generation technology	**Installed capacity (*GW*)**	**Price (*£/MWh*)**
CCGT/OCGT/CHP/GAS	27.039	Dependant on gas price
Coal /Dual fuel	28.861	32
Nuclear	11.984	0 (must run)
Biomass	0.012	58
Wind	1.839	0 (must run)
Hydro	1.446	0 (must run)
Oil/diesel	4.035	60
Interconnector	1.988	70
Tidal	0.007	68
Thermal	0.02	40
Waste	0.018	69
Pumped storage	1.796	70
	Total: 79.045	

Capacity factor for wind and hydro at 30%

Thermal efficiencies for the following generating units:

CCGT = 55%, OCGT = 39%, CHP = 55%, GAS = 39%

Table 12. Cost of shedding

Load	**Cost**
Electrical load shedding	2000(*£/MWh*) [26]
Gas load shedding	11.1 (*£/cm*) [27]

Table 13. Gas storage data

Storage facility	**Storage Type**	**Max withdrawal rate (*mcm/d*)**	**Max injection rate (*mcm/d*)**	**Working capacity (*mcm*)**	**Cushion capacity (*mcm*)**
Avonmouth	LNG	14.37	0.2116	80.6	0.01
Dynevor Arms	LNG	4.5264	0.2392	28	0.01
Glenmavis	LNG	9.3	0.4232	46.46	0.01
Partington	LNG	20.22	0.2208	103.2	0.01
Rough	Depleted gas field	44	15	3340	5200
Hatfield Moor	Depleted gas field	2.4	2.4	116	174
Humbly Grove	Depleted gas field	7.5	8.5	280	420
Hornsea	Salt cavern	18.5	2	320	158
Hole House	Salt cavern	2.8	5.6	25	12.3

Table 14. Gas storage costs

Storage facility	**Withdrawal costs (*£/cm*)**	**Injection costs (*£/cm*)**
Avonmouth	0.004	0.0048
Dynevor Arms	0.003	0.0054
Glenmavis	0.003	0.03
Partington	0.004	0.061
Rough	0.011	0.109
Hatfield Moor	0.005	0.054
Humbly Grove	0.005	0.054
Hornsea	0.005	0.054
Hole House	0.005	0.054

Table 15. Gas terminal price data for December 2005

Time (days)	**Gas price (*£/cm*)**
1	0.377
2	0.39
3	0.389
4	0.387
5	0.339
6	0.346
7	0.324
8	0.285
9	0.274
10	0.268
11	0.285
12	0.324
13	0.34
14	0.292
15	0.294
16	0.297
17	0.303
18	0.302
19	0.255
20	0.263
21	0.257
22	0.25
23	0.145
24	0.115
25	0.146
26	0.131
27	0.258
28	0.27
29	0.39
30	0.243
31	0.232

Gas prices 5% lower at the St. Fergus terminal

Table 16. Bus generation data

Bus	Installed GB generation capacity (*MW*)											
	CCGT / OGT / CHP	Wind	Coal / Duel fuel	Hydro	Oil / Diesel	Nuclear	Biomass	waste	Tidal	Pumped storage	Inerconn ector	Thermal
1	-	422	-	831	44	-	-	9	7	-	-	20
2	1554	419	-	-	-	-	-	-	-	-	-	-
3	-	86	-	256	12	-	-	-	-	-	-	-
4	-	325	-	236	-	-	-	-	-	-	-	-
5	-	48	2304	-	-	-	-	-	-	440	-	-
6	156	539	1152	123	-	2490	12	9	-	-	-	-
7	2030	-	756	-	-	1207	-	-	-	-	-	-
8	-	-		-	-	-	-	-	-	-	-	-
9	3042	-	1995	-	-	3412	-	-	-	1356	-	-
10	4813	-	7874	-	-	-	-	-	-	-	-	-
11	-	-		-	-	-	-	-	-	-	-	-
12	232	-	4024	-	-	-	-	-	-	-	-	-
13	2958	-	4004	-	-	-	-	-	-	-	-	-
14	3454	-		-	1475	-	-	-	-	-	-	-
15	3929	-	3641	-	1034	1731	-	-	-	-	-	-
16	4871	-	3111	-	1470	3144	-	-	-	-	1988	-

Table 17. Line data

Line	Maximum transmission capacity (*MW*)
TB1	400
TB2	1620
TB3	220
TB4	1520
TB5	2550
TB6	2200
TB7	3060
TB8	1661
TB9	5761
TB10	10603
TB11	5974
TB12	3957
TB13	11551
TB14	5174
TB15	6423

Table 18. Compressor data

Item	Lower bound	Upper bound
Compressor pressure ratio	1	4

For each compressor: Efficiency = 0.8, Polytropic exponent = 1.27, Beta = 0.0002, Maximum Power = 50MW

Table 19. Gas terminal capacities

Gas terminal	**Maximum Supply capacities (*mcm/d*)**
St. Fergus	117.9
Teesside	30.6
Barrow	27
Point Of Ayr	4.5
Easington	15.3
Theddlethorpe	27
Bacton	108.9
Isle Of Grain (IOG)	15.3

For each pipe: Efficiency (E) = 0.92, Temp = 288K, Min pressure = 38 bars, Max pressure = 85 bars, Compressibility factor (Z) = 0.95

Appendix IV: GB gas and electricity network

Figures 12 and 13 show the network flows at day 26 for case A (GB reference).

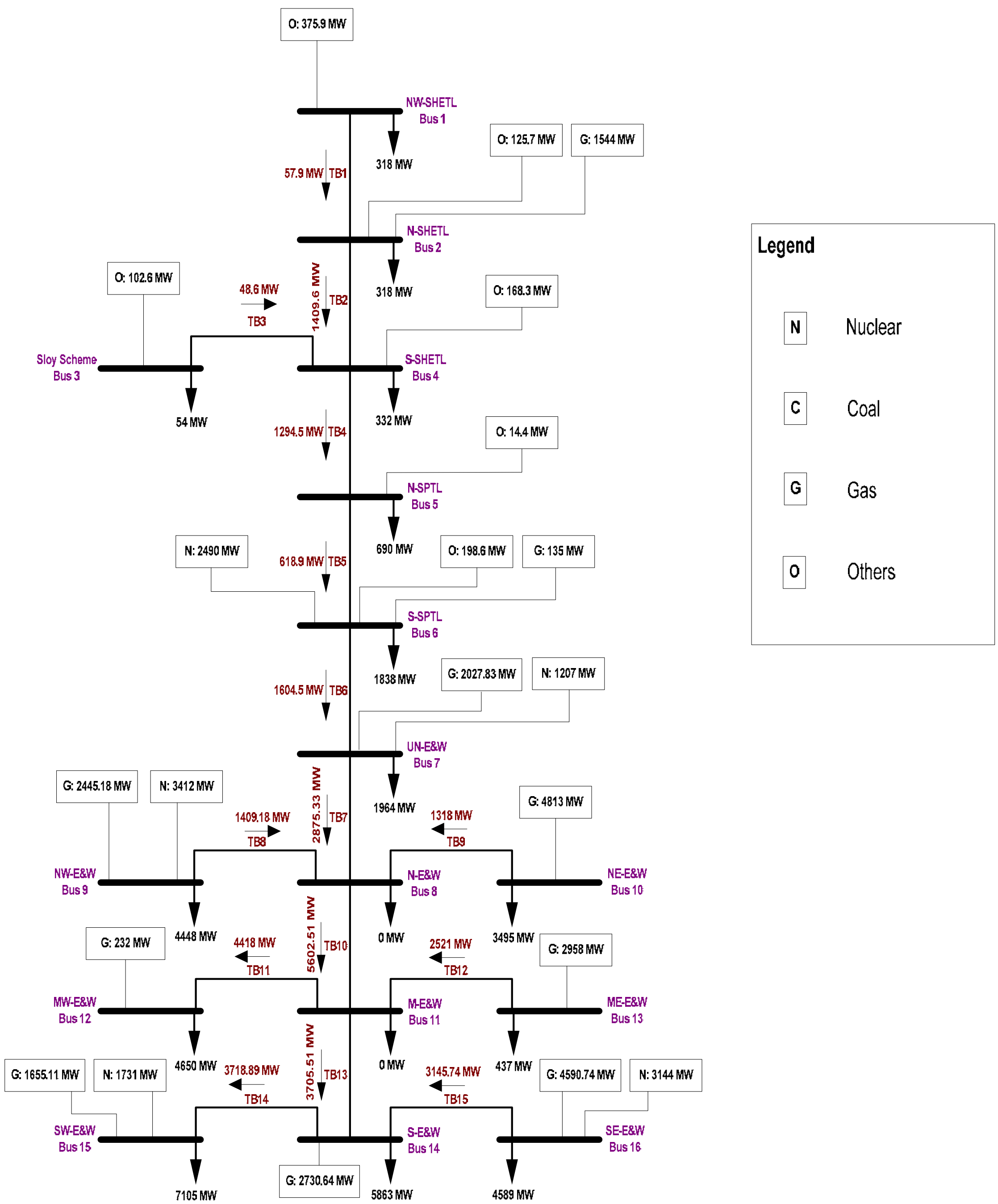

Figure 12. GB electricity flows for day 26

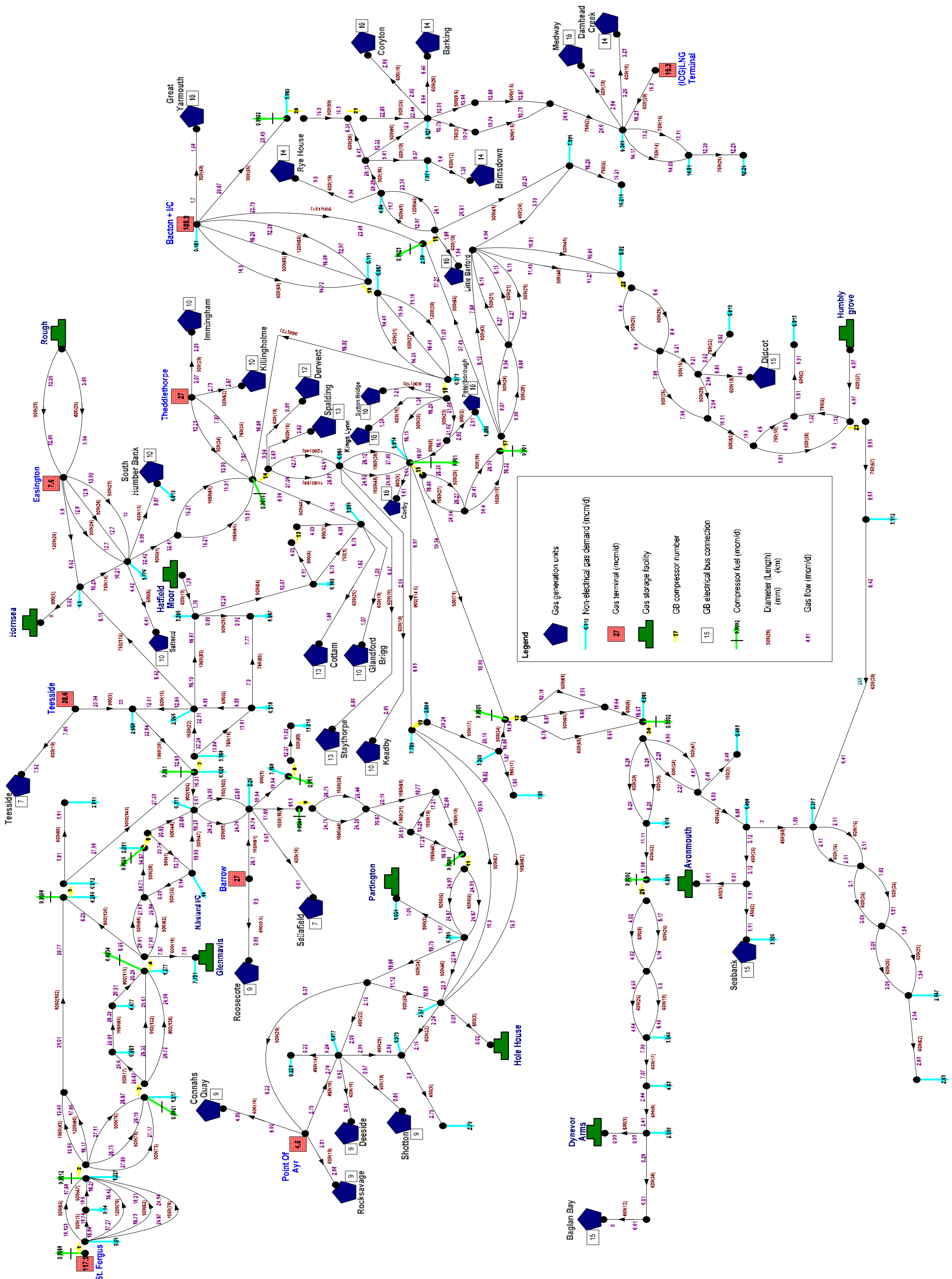

Figure 13. GB gas flows for day 26